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THE FUNDAMENTAL LAWS OF ARITHMETIC: PSYCHOLOGICAL LOGIC.¹

[This article on psychological logic continues the translation of the Preface of Professor Frege's *Grundgesetze*, of which the first part was published in *The Monist* of October, 1915. This part is divided from the first part by the note: "Mathematicians who do not care to study the mistakes of philosophy are recommended to break off here their reading of the Preface."]

FROM the leading current presentations of logic I cannot hope for approval of the distinction that I make between the *characteristic* (*Merkmal*) of a concept and the *property* (*Eigenschaft*) of an object,² for these presentations seem to be thoroughly infected by psychology. If we consider instead of things themselves their subjective images—our own notions or presentations (*Vorstellungen*)—all the more delicate distinctions in the things themselves naturally are lost, and others appear instead which are logically quite worthless. And this brings me to what I have to say regarding the factors which prevent my book from having an effect on logicians. It is the injurious invasion of logic by psychology. The conception of logical laws must be the decisive factor in the treatment of logic, and that conception depends upon what we understand by the word "true." It is generally admitted at the very beginning that logical laws must be rules of conduct to guide

¹ [Translated by Johann Stachelroth and Philip E. B. Jourdain.]

² In the *Logik* of Benno Erdmann I find no trace of this important distinction.

thought to truth; but this admission is only too easily forgotten. The double meaning of the word "law" is here fatal to clearness of thought with most people. In one sense law says what is, and in the other it dictates what must be. Logical laws can only be called "laws of thought" in the latter sense; they lay down how we must think. Every law that says what is may be understood as dictating that we are to think in conformity with it and is in this sense a law of thought. This holds good in geometry and physics as well as in logic. Logical laws deserve the name "laws of thought" with more right than these other scientific laws only if we wish to express by the name that they are the *most general* laws in that they dictate how we must think whenever we think at all. But the phrase "law of thought" leads people to believe that thinking is governed by these laws in the same way that laws of nature govern events in the world around us. In that case they would be psychological laws, for thinking is a psychical process. But if logic had to deal with psychological laws it would be a part of psychology; and indeed it is sometimes viewed as a part of psychology. These laws of thought are in that case looked upon as rules of conduct in the sense that they indicate the average; just as we may say that healthy digestion takes place in man, or that people speak grammatically, or that people dress in fashion. And then we can only say that what men believe on the average to be true thought, at present and as far as human beings are known to us, is conducted according to these laws. Consequently if we wish to agree with the average man we must think according to these laws. But just as what is modern to-day will not be modern after some time and even now is not modern with the Chinese, we can set up psychological laws of thought only under limitations of space and time. Such would be the case if logic were concerned with the growth of our opinions on truth and not with truth itself.

These two matters are what psychologizing logicians confuse with one another. Thus Benno Erdmann, in his *Logik*,³ defines truth as general validity (*Allgemeingültigkeit*), founds this general validity on general certainty of the object about which we judge (*Allgemeingewissheit des Gegenstandes, von dem geurtheilt wird*), and founds this certainty on the general agreement of those who judge. Thus the truth is finally reduced to what the individual supposes to be true. To that I can only reply that the fact of being true (*Wahrsein*) is different from the fact of being regarded as true (*Fürwahrgehaltenwerden*), that it does not matter whether the fact is so regarded by one or by many or by everybody, and that what is true cannot be reduced to it. There is no contradiction in something being true though everybody thinks it to be false. I do not understand by "logical laws" psychological laws of belief, but laws of truth. If it is true that I write this in my room on July 13, 1893, while the wind is howling outside, it remains true even though everybody think it false. If, thus, being true is independent of the truth being acknowledged by some one, then the laws of truth are not psychological laws, but boundary stones on an eternal foundation which may be inundated by our thought but are not movable. And because they are immovable they are important for our thought if it wishes to get at the truth. They do not stand in the same relation to thought as grammatical laws to language; they express the essence of our thought and change with it.

Quite different from this is Erdmann's conception of logical laws. He doubts their absolute and eternal validity, and wishes to limit them to our thought as it is now.⁴ "Our thought," I suppose, can only be the thought of human

³ Halle a. S., 1892, Vol. I, *Logische Elementarlehre*, pp. 272-275. [A second edition of this volume appeared in 1907.]

⁴ *Ibid.*, pp. 375 ff.

beings known up to the present time. For there would remain the possibility of discovering human or other beings who could execute sentences contrary to our laws. Suppose that were to happen, then Erdmann would say: "Here is the proof that those fundamental laws do not hold good everywhere." If they are psychological laws, their verbal expression must make known the species of beings whose thought is found by experience to be governed by them. I would say: "There are therefore beings which do not immediately recognize certain truths as we do, but which are perhaps obliged to take the lengthier road of induction." But what if beings were found whose laws of thought were absolutely in opposition to ours and consequently often led in applications to contrary results? The psychological logician could only assent to the fact, and say: "With them the former laws apply and with us the latter." I would say: "Here we have a hitherto unknown kind of madness." Whoever understands by "logical laws" laws which dictate how thought must be guided, i. e., laws of truth and not natural laws of human belief, will ask: "Who is right? Whose laws are in accord with the laws of truth?" The psychological logician cannot ask such a question; for it would mean that he recognized laws of truth which are not psychological. It is hardly possible to falsify the meaning of the word "true" more grossly than by referring to the judge. It is not to the point to object that the remark, "I am hungry," may be true for one and false for another. The remark may be so, but not the thought, for the word "I" in the mouth of another refers to a different being, and therefore the above remark in another person's mouth expresses a different thought. All determinations of place, of time, and so on, belong to the thought whose truth is in question; the truth itself is not subject to place or time. How then does the principle of identity run? Shall we say thus: "In 1893 it is impossible

for human beings to admit that an object may be different from itself"; or thus: "Every object is identical with itself"? The former law treats of human beings and contains a determination of time; in the latter law there is no mention of human beings nor of a time. The latter is a law of truth; the former is one of human belief. Their contents are altogether different, and they are independent of each other, so that one cannot be deduced from the other. It is therefore very confusing to designate both by the same name, "principle of identity." Such mixtures of altogether different things are the cause of the awful confusion which we meet in the doctrines of psychological logicians.

The question why and with what right we recognize a logical law to be true, logic can only answer by reducing it to other logical laws. Where that is not possible, logic can give no answer. Leaving aside logic for a moment, we may say that we are obliged by our nature and outer circumstances to form judgments, and if we judge we cannot reject this law—the law of identity for instance. We must recognize it if we do not wish to bewilder our thought and at last abandon all judgment. I will neither dispute nor try to confirm this opinion, and will merely observe that we have here no logical implication. There is given merely a ground for supposing it to be true and not for its being true. Moreover the fact that we find it impossible to reject the law spoken of does not prevent us from supposing the existence of beings who reject it; but it prevents our supposing that their views are correct with regard to that point. It also prevents our doubting whether we or they are right. That at least is true of myself. If others should dare to accept and to doubt a law in the same breath, they would give me the same impression as if they were trying to jump out of their skins, and I would urgently warn them against such an attempt. He who has once accepted a law of truth

has, by that very fact, accepted a law which dictates how a judgment is to be made, no matter where or when or by whom.

If I review the whole matter it seems to me that different conceptions of the truth are the origin of the controversy. I look upon truth as something objective and independent of the person who judges. It is not so according to the psychological logicians. What Erdmann calls "objective certainty" is only a general acknowledgment proceeding from those who judge, and which therefore is not independent of them but may change with their psychical nature.

We can generalize this still more. I acknowledge an objective domain which is not a domain of actual things; while the psychological logicians, without more ado, look upon the non-actual as subjective. And yet it is impossible to see why that which has a value independent of the person who judges must be actual, that is to say must be capable of a direct or indirect action upon the senses. Such a connection between the ideas is not to be discovered. We can even quote examples which show the contrary. The number 1, for instance, cannot easily be thought actual, unless indeed we are disciples of John Stuart Mill. It is impossible, on the other hand, to assign to each person his number 1; for then we would have to inquire how far the property of these units agrees. And if one person says "once 1 is 1," and another "once 1 is 2," we could only state the difference and say: "Your 1 has one property and mine another." There could be no question of a quarrel as to who was right nor of making an attempt to teach; since there is no common object. Evidently this is quite contrary to the meaning of "1" and the sentence "once 1 is 1." Since the number 1, as being the same for everybody, appears to everybody in the same manner, it can no more be investigated by psychological observations than the

moon can. Whatever notions there may be of the number 1 in different minds, they must be distinguished from the number 1 just as the notions of the moon must be from the moon itself. Because the psychological logicians deny the possibility of the objective non-actual, they suppose concepts (*Begriffe*) to be notions or presentations (*Vorstellungen*) and assign them to psychology. But the weight of truth is too great for this to be easily practicable. And hence comes an indefiniteness in the use of the word "notion;" at times it seems to denote something which belongs to the psychical life of each separate individual and which amalgamates with other notions and associates with them according to psychological laws; and at times it seems to denote something which faces everybody in the same way and in which a person who has a notion of it is neither mentioned nor even tacitly supposed. These two kinds of usage of the word are incompatible with each other; for those associations and amalgamations only occur in particular individuals and with something that belongs to these individuals particularly, such as their joy and pain. We ought never to forget that the notions of different individuals, no matter how much they resemble one another,—and this, by the way, we are unable to ascertain satisfactorily,—do not coincide but are to be distinguished from one another. Every one has notions of his own which are not those of others. Here of course I understand "notion" in a psychological sense. The indeterminate use of this word causes confusion and helps the psychological logicians to hide their weakness.

When will this confusion stop? Everything is finally drawn into psychology; the boundary line between objective and subjective disappears more and more, and even actual objects are treated psychologically as notions. For what is *actual* but a predicate? And what are logical predicates but notions? Thus everything drifts into idealism

and then quite logically into solipsism. If every one denoted something different by the word "moon," namely one of his notions, much in the same way that he would express his pain by the exclamation "oh," the psychological manner of consideration would of course be justified; but a dispute about the properties of the moon would be to no purpose. One person might quite well assert the contrary about his moon to what another person could say with the same right of his. If we could not grasp anything else but what is in ourselves, a conflict of opinions and a mutual understanding would be alike impossible, because there would be no common ground; and such a common ground cannot be formed by a notion in the sense of psychology. There would be no logic capable of being judge in a dispute of opinions. That I may not seem to be fighting against windmills, I will take a definite book and show in it the inevitable sinking of psychological logic into idealism. I choose for that the above-mentioned logic of Erdmann, as it is one of the most recent works of the psychological school and is not likely to be denied all importance. Let us look at the following sentence:⁶

"Thus psychology teaches with certainty that the objects of recollection and imagination as well as those of the notions of morbid hallucinations and illusions are of an ideal nature. . . . Furthermore the whole realm of mathematical notions properly so called, from the series of numbers to the objects of mechanics, are ideal.

In this strange collection the number 10 is actually put on the same level as hallucinations. Here evidently that which is objective and not actual is mixed up with what is subjective. Some objective things are actual, others not. *Actual* is only one of many predicates, and has no more to do with logic than the predicate *algebraic* has to do with a curve. Of course through this confusion Erdmann gets

⁶ *Ibid.*, Vol. I, p. 85.

mixed up with metaphysics, however much he tries to keep away from it. I hold it to be a sure sign of error if logic needs metaphysics and psychology, sciences which themselves must have a foundation of logical propositions. Where then is the ultimate basis upon which everything rests? Or is Erdmann's case similar to that of Münchhausen who pulled himself out of the mire by his own hair? I doubt very much the possibility of this Münchhausen-like process even in logic, and suspect that Erdmann will remain in the mire of psychological metaphysics.

There is no objectivity, properly speaking, for Erdmann; for everything is notional with him. Let us convince ourselves of this by his own statements:

"To form a relation between things of which we have notions (*Vorgestelltes*) the judgment needs at least two points of reference between which it takes up its position. As a *statement* about notions (*Vorgestelltes*), it demands that one of these points of reference should be determined as the thing (*Gegenstand*) about which the statement is made, the subject, and the second as the thing which is stated, the predicate."⁶

We see here first of all that both the subject of which something is said, and the predicate, are called objective things (*Gegenstände*) or things of which we have notions (*Vorgestelltes*). Instead of "the thing" might better have been written "the thing of which we have a notion"; for we read: "For things are things of which we have notions."⁷ But *vice versa* everything of which we have a notion is also to be a thing: "According to its origin a thing of which we have a notion is either, on the one hand, an object of sense-perception or of self-consciousness, and on the other hand it is either original or derived."⁸ What arises from sense-perception or from self-consciousness is certainly of a psychological nature. Things, things of which we have

⁶ *Ibid.*, p. 187.

⁷ *Ibid.*, p. 81.

⁸ *Ibid.*, p. 38.

notions, and hence subject and predicate, are thus assigned to psychology. This is confirmed by the following passage:

"That of which we have a notion (*das Vorgestellte*) and the notion (*die Vorstellung*) are the same thing: the thing of which we have a notion is the notion, and the notion is the thing of which we have a notion."⁹

The word "notion" is usually taken in a psychological sense; that that is also Erdmann's custom we can see from the following passages: "Consciousness is thus the general concept where corresponding particulars are forming notions and willing";¹⁰ "the forming of notions is composed of notions. . . .and the courses of notions (*Vorstellungsverläufen*)."¹¹ After this we must not be surprised that an object comes into existence in a psychological way: "As far as a mass of perceptions. . . .offers the same to former excitations and to the stimuli released by them, it *reproduces* the residues of memory which descend from that sameness of former excitations and *amalgamates* with them to form the object of the apperceived notion."¹² On the next page is then shown, as an example, how a steel-engraving of Raphael's Sistine Madonna is made in a psychological way without steel-plate, ink, press, or paper.

After all this there can be no possible doubt but that the object spoken of, the subject, is supposed by Erdmann to be a notion in the psychological sense of the word, as is also the predicate, the thing which is said. If that were so we would never be able to say truthfully of an object that it is green; for there are no green notions. Neither would I be able to say of a subject that it is independent of having a notion formed of it, or of myself as one who forms a notion of it, any more than that my decisions are independent of my will and of myself, the willer; but they would be destroyed with me if I were destroyed. In conse-

⁹ *Ibid.*, pp. 147-148.

¹¹ *Ibid.*, p. 36.

¹⁰ *Ibid.*, p. 35.

¹² *Ibid.*, p. 42.

quence of this there is no objectivity proper for Erdmann, as also results from the fact that he posits the thing of which we have a notion or the notion in general—the *thing* in the most general sense of the word—as the *summum genus*.¹⁸ He is therefore an idealist. If the idealists thought logically they would consider the sentence, “Charlemagne conquered the Saxons,” to be neither true nor false, but a fiction, just as we are in the habit of considering, say, the sentence, “Nessus carried Deianeira across the river Euenus.” For the sentence, “Nessus did not carry Deianeira over the the river Euenus,” could only be true, if the name “Nessus” was borne by somebody. It would not be easy to move the idealist from this point of view. But we need not put up with the falsification of the meaning of the sentence arising from assuming that I wanted to say something about my notions when I spoke about Charlemagne; I simply meant to indicate a man independent of myself and my notions and to make a statement about him. We may grant to the idealists that the attainment of this intention is not quite certain and that perhaps in my attempt I may stray unintentionally from the truth into fiction. But by that nothing can be changed in the sense. By the sentence, “This blade of grass is green,” I express nothing about my notions; I indicate none of my notions by the words, “This blade of grass.” If I did so the sentence would be false. There now enters a second falsification, i. e., that my notion of green is expressed by my notion of this blade of grass. I repeat: In this sentence there is no question whatever about my notions; that meaning is wholly due to the idealists. By the way, I fail absolutely to understand how a notion of something can be expressed. It would be just such a falsification to say that in the sentence, “The moon is independent of myself and my forming notions,” my notion of independence of myself

¹⁸ *Ibid.*, p. 147.

and my forming notions would be expressed of my notion of the moon. With that the objectivity proper would surely be rejected and something quite different put into its place. No doubt it is possible that, in judging, such a play of notions occurs; but that is not the meaning of the sentence. Observe that with the same sentence and the same meaning of the sentence the play of notions may be quite different. And it is this logically indifferent accompanying appearance that our psychological logicians take for the real object of research.

As may be easily understood, the nature of the matter militates against any sinking into the mire of idealism, and Erdmann would not like to admit that for him there is no objectivity proper. But we can just as easily understand the fruitlessness of this effort. For if all subjects and all predicates are notions, and if all thinking is nothing but the producing, connecting, and changing of notions, it is impossible to conceive how on earth we arrive at anything objective. A sign of this vain struggle is the use of the words, "a thing of which we have a notion," and "an objective thing," which at first might seem intended to indicate something objective in opposition to notions. But the words only *seem* to do this, for we have seen that they denote the same thing. Why then this superfluity of expression? That is not difficult to guess. We must notice that an object of our notions is spoken of, though the object is supposed to be itself a notion. Thus it would be a notion of a notion. What relation of notions is meant by this? Though this is obscure, it is quite comprehensible how, from the conflict between the nature of the matter and idealism, such whirlpools can arise. Everywhere here we see the object of which I make a notion for myself confused with the notion, and then we see the difference stand out again. We see this conflict also in the following sentence: "For a notion whose object is general is therefore, as such,

as a process of consciousness, as far from being general as a notion is itself real because its object is posited as real; or as an object which we perceive. . . . to be sweet is given by notions which are themselves sweet."¹⁴ Here the real truth asserts itself. I could almost agree with that. But if we notice that according to Erdmann's principles the objects of which we have notions and the objects which are given by notions are notions themselves, we see that all our straining for agreement must be in vain. I also beg my readers to remember the words "as such" which appear similarly just before the last passage: "Where the actuality of a thing is stated, the subject of this judgment is not the object or the thing as such of which we have a notion, but on the contrary the *transcendental* which is presupposed as the basis of the being of this 'thing of which we have a notion' and is represented in the latter. The transcendental is not to be looked upon as the unknowable. . . . but its transcendence is only to consist in the independence of the process of forming a notion (*Vorgestelltwerden*)."¹⁵

This is merely another vain attempt to work himself out of the mire. If we take the words seriously, they form a statement that in this case the subject is not a notion. But if this is possible it is not clear why with other predicates which indicate special modes of activity or actuality the subject must be a notion,—for instance in the judgment, "the earth is magnetic." And so we would arrive at the opinion that only in a few judgments the subject is a notion. But if it is once admitted that it is not essential for either the subject or the predicate to be a notion, the foundation is pulled away from underneath the whole of psychological logic. All the psychological considerations of which our books on logic are full just now turn out then to be irrelevant.

But I dare say we must not take the transcendence with

¹⁴ *Ibid.*, p. 86.

¹⁵ *Ibid.*, p. 83.

Erdmann quite seriously. I only need to remind him of his declaration:¹⁸ "The *metaphysical* limit of our ideation (*Vorstellens*), the transcendental, is also subordinate to the *summum genus*," and with that he founders; for his *summum genus* is what we have a notion of, or is notion in general. Or should the above word "transcendental" be used in a different sense? In every case, we are to think, the transcendental is subordinate to the *summum genus*.

Let us reflect a little about the expression "as such." I will suppose that somebody wants to make me imagine that all objects are nothing but pictures on the retina of my eye. Very well, I have no objection to make so far. But now he asserts that the tower is bigger than the window through which I suppose that I am seeing it. To this I would say: "Either the tower and the window are not both pictures on the retina of my eye—in which case the tower may be bigger than the window; or the tower and the window are, as you say, pictures on my retina—in which case the tower is not bigger but smaller than the window." Now he tries to extricate himself from the dilemma by using the words "as such," and says: "The picture of the tower on the retina as such is indeed not bigger than that of the window." At this point I would almost like to jump out of my skin and shout at him: "Well then, the picture of the tower on the retina is not bigger than that of the window, and if the tower were the picture of the tower and the window the picture of the window, then the tower would not be bigger than the window, and if your logic teaches you differently it is not worth anything." This "as such" is an excellent discovery for writers who are not clear in their statements and who do not want to say either yes or no. But I am not going to put up with this hovering between the two, and I ask: "If actuality is predicated of a thing, is the subject of the judgment the notion? An-

¹⁸ *Ibid.*, p. 148.

swer me yes or no." If it is not, then it is, I suppose, the transcendental which is presumed to be the basis of the being of this notion. But this transcendental is itself notional (*Vorgestelltes oder Vorstellung*). Thus we are driven to the supposition that the ideated transcendental is not the subject of the judgment but the transcendental which is presumed as the basis of the being of this ideated transcendental. Thus we would have to keep on continually, but however far we go we never should get out of the subjective. We might begin the same game with the predicate, and not necessarily with the predicate *actual* but, say, with *sweet*. We would then say: "If we speak about the actuality or the sweetness of a thing the predicate is not the ideated actuality or sweetness but the transcendental which is supposed to be the basis of the being of this ideation." But we could not rest there; we would be driven on and on. What can we learn from this? That psychological logic is mistaken if it thinks that the subject and predicate of judgments are notions in the sense of psychology, and that psychological considerations are as out of place in logic as in astronomy or geology. If we wish to get out of the subjective, we have to conceive knowing as an activity which does not create what is known but which grasps what already exists. The illustration of grasping is very well fitted to elucidate the matter. If I grasp a pencil, many different things take place in my body: excitations of the nerves, changes of the tension and the pressure of the muscles, sinews, and bones, and changing of the motion of the blood. But the totality of these processes is neither the pencil nor does it create the pencil. The pencil exists independently of these processes, and it is essential for the fact of grasping that there is something to be grasped; it is not our internal changes which alone make up the grasping. In the same way, what we grasp mentally is independent of those notions and their changes that be-

long to or accompany this grasping. What we grasp is neither the totality of these processes nor is it created by this totality as part of our psychical life.

Let us now see how the finer distinctions in the subject-matter of logic become obliterated in psychological logic. This has already been referred to above when we spoke of *characteristic* and *property*. With this is connected the distinction of *thing* or *object* (*Gegenstand*) and *concept* (*Begriff*) emphasized by myself, and that of concepts of the *first and second stage* (*Stufe*). These distinctions are of course indiscernible to the psychological logician; with such logicians everything is just notion. They have not got the right conception of those judgments which we express by "there is." This existence is confused by Erdmann¹⁷ with actuality, which, as we have seen, is not clearly distinguished from objectivity. Of what things do we assert that it is actual when we say that there are square-roots of 4? Is it 2 or -2 ? But neither the one nor the other is mentioned here in any way. And if I were to say that the number 2 acts or is active or actual, it would be false and quite different from what I mean by the sentence "there are square-roots of 4." The confusion here under consideration is nearly the grossest possible; for it is not one between concepts of the same stage, but a concept of the first stage is confused with one of the second stage. This is characteristic of the dullness of psychological logic. When we have arrived at a somewhat broader standpoint we may be surprised that such a mistake could be made by a professional logician; but we must have grasped the distinction between concepts of the first and second stages before we can estimate the magnitude of the error spoken of, and psychological logic cannot do that. Here what most stands in the way of psychological logic is that its exponents think such a lot of psychological depth, which is

¹⁷ *Ibid.*, p. 311.

after all nothing but a psychological falsification of logic. And that is how our thick books of logic come to be; they are puffed out with unhealthy psychological fat which conceals all finer forms. Thus a fruitful collaboration of mathematicians and logicians is made impossible. While the mathematician defines objects, concepts, and relations, the psychological logician watches the becoming and changing of notions, and at bottom the defining of the mathematician must appear only foolish to him because it does not reproduce the essence of ideation. He looks in his psychological *camera obscura* and says to the mathematician: "I cannot see anything at all of what you are defining." And the mathematician can only reply: "No wonder, for it is not where you are looking for it."

This may be enough to put my logical standpoint, by way of contrast, into a clearer light. The distance between my point of view and that of psychological logic seems to me so very great that there is no prospect of my having at present any influence through my book upon psychological logic. It seems to me that the tree planted by me would have to lift an enormous weight of stone in order to gain room and light for itself. Nevertheless I would not like to give up all hope that my book may later on help to overthrow psychological logic. As a step toward this end, my book will not, I hope, be quite unnoticed by mathematicians, so that mathematicians will have to come to terms with it. And I believe that I may expect some help from that quarter; for mathematicians have at bottom a common cause with me against the psychological logicians. As soon as mathematicians condescend to occupy themselves seriously with my book, if only to disprove it, I believe I have won. For the whole of the second part is really a test of my logical convictions. It is improbable that such an edifice could be erected upon an unsound base. Those who have other convictions have only to try to erect a similar construction

upon them, and they will soon be convinced that it is not possible or at least is not easy. As a proof of the contrary, I can only admit the production by some one of an actual demonstration that upon other fundamental convictions a better and more durable edifice can be erected, or the demonstration by some one that my premises lead to manifestly false conclusions. But nobody will be able to do that. May my book then, even though it comes rather late, contribute to a revival of logic.

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